

CAPACITORS

LAB ELEC3.COMP

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INTRODUCTION

The charge q on a capacitor's plate is proportional to the potential difference V across the capacitor. We express this relationship with

$$V = \frac{q}{C},$$

where C is a proportionality constant known as the *capacitance*. Capacitance is a measure of a capacitor's ability to store charge. C is measured in the unit of the farad, F, (1 farad = 1 coulomb/volt).

If a capacitor of capacitance C (in farads) is initially charged to a potential V_0 (volts) with the switch in position 32 of *Figure 1* and then the switch is put in position 34, a time-dependent current will flow across a resistor R (in ohms), according to Ohm's law. This is known as an RC (resistor-capacitor) circuit as represented by the figure.

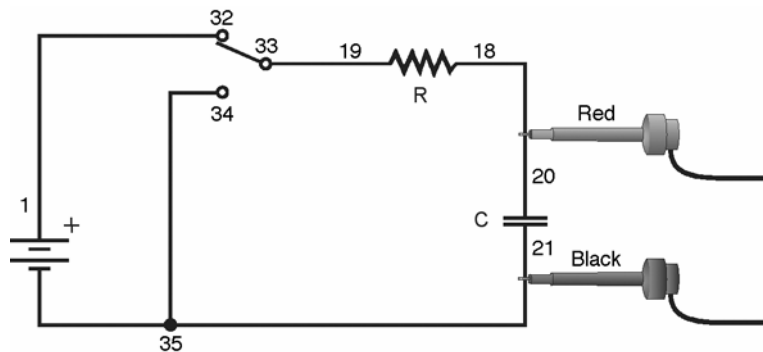


Figure 1

As the current flows, the charge q is depleted, reducing the potential across the capacitor, which in turn reduces the current. This process creates an exponentially decreasing current, modeled by

$$V(t) = V_0 e^{-\frac{t}{RC}}.$$

The rate of the decrease is determined by the product RC , known as the *time constant* of the circuit. A large time constant means that the capacitor will discharge slowly accordingly.

When the capacitor is charged, the potential across it approaches the final value exponentially, modeled by

$$V(t) = V_0 \left(1 - e^{-\frac{t}{RC}} \right).$$

The same time constant RC describes the rate of charging as well as the rate of discharging.

Another consideration regarding capacitors is the storage of energy with respect to the charge or the potential difference.

We can calculate the potential energy U of the capacitor by calculating the work W required to charge it. With q and v , respectively, the charge and potential difference, then we have the relationship $v=q/C$. The work done dw to transfer an additional element of charge is

$$dw = v dq = \frac{q dq}{C}.$$

The total work w needed to increase the capacitor charge q from zero to final value Q is

$$W = \int_0^Q dw = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}.$$

This is also equal to the work done by the electric field on the charge in the capacitor discharging. In the discharging q decreases from Q to zero as the dq element of charge “falls” through a potential difference varying from V to zero.

With the potential energy of the uncharged capacitor as zero, then the potential energy U stored in the charged capacitor is equal to the work done in charging it, that is

$$U = Q^2/2C = CV^2/2 = QV/2$$

where Q is in coulombs, V is in volts and C in farads; then U is in joules.

PURPOSE

To Measure an experimental time constant of a resistor-capacitor circuit and compare its value to that predicted from the component values of the resistance and capacitance. To also measure the potential across a capacitor as a function of time as it discharges and as it charges and fit an exponential function to the data noting that one of the fit parameters corresponds to an experimental time constant.

MATERIALS


Computer
Vernier computer interface
Logger *Pro*
Vernier Voltage Probe

Vernier Circuit Board with batteries, or
10 μF non-polarized capacitor
100 $\text{k}\Omega$, 47 $\text{k}\Omega$ resistors
Two C or D cells with battery holder
Single-pole, double-throw switch
Connecting wires

PRELIMINARY QUESTIONS

1. Consider a candy jar, initially with 1000 candies. You walk past it once each hour. Since you don't want anyone to notice that you're taking candy, each time you take 10% of the candies remaining in the jar. Sketch a graph of the number of candies for a few hours.
2. How would the graph change if instead of removing 10% of the candies, you removed 20%? Sketch your new graph.

PROCEDURE

1. Connect the circuit as shown in Figure 1 above with the 10 μF capacitor and the 100 $\text{k}\Omega$ resistor. Record the values of your resistor and capacitor in your Data Table, as well as any tolerance values marked on them. If you are using the Vernier Circuit Board, the terminal numbers are shown on the schematic to help you to connect the circuit.
2. Connect the Voltage Probe to Channel 1 of the computer interface, as well as across the capacitor, with the red (positive lead) to the side of the capacitor connected to the resistor. Connect the black lead to the other side of the capacitor.
3. Open the file in the "27 Capacitors" file in the *Physics with Computers* folder.
4. Charge the capacitor for 30 s or so with the switch in the position as illustrated in Figure 1 (position 32). You can watch the voltage reading at the bottom of the screen to see if the potential is still increasing. Wait until the potential is constant.
5. Click to begin data collection. As soon as graphing starts, throw the switch to its other position (position 34) to discharge the capacitor. Your data should then show a decreasing function.
6. To compare your data to the model, select only the data after the potential has started to decrease by dragging across the graph; that is, omit any initial constant portion. Click the Curve Fit button , and from the function selection box, choose the Natural Exponent function, $A \cdot \exp(-Ct) + B$. Click , and inspect the fit. Click to return to the main graph window.


7. Record the value of the fit parameters in your Data Table. Notice that the C used in the curve fit is not the same as the C used to stand for capacitance. Compare the fit equation to the mathematical model for a capacitor discharge proposed in the introduction,

$$V(t) = V_0 e^{-\frac{t}{RC}}$$

How is fit constant C related to the time constant of the circuit, which was defined in the introduction?

8. Print or sketch the graph of potential vs. time. Choose Store Latest Run from the Experiment menu to store your data. You will need these data for later analysis.
9. The capacitor is now discharged. To monitor the charging process, click . As soon as data collection begins, throw the switch the other way. Allow the data collection to run to completion.
10. This time you will compare your data to the mathematical model for a capacitor charging,

$$V(t) = V_0 \left[1 - e^{-\frac{t}{RC}} \right]$$

Select the data beginning *after* the potential has started to increase by dragging across the graph. Click the Curve Fit button, , and from the function selection box, choose the Inverse Exponent function, $A*(1 - \exp(-Ct)) + B$. Click and inspect the fit. Click to return to the main graph.

11. Record the value of the fit parameters in your Data Table. Compare the fit equation to the mathematical model for a charging capacitor.
12. Hide your first runs by choosing Hide Data Set from the Data menu. Remove any remaining fit information by clicking the upper left corner in the floating boxes.
13. Now you will repeat the experiment with a resistor of lower value. How do you think this change will affect the way the capacitor discharges? Rebuild your circuit using the 47 kΩ resistor and repeat Steps 4 – 11.

DATA TABLE

Trial	Fit parameters				Resistor	Capacitor	Time constant
	A	B	C	1/C	R (Ω)	C (F)	RC (s)
Discharge 1							
Charge 1							
Discharge 2							
Charge 2							

ANALYSIS

1. In the Data Table, calculate the time constant of the circuit used; that is, the product of resistance in ohms and capacitance in farads. (Note that $1\ \Omega\ F = 1\ s$).
2. Calculate and enter in the Data Table the inverse of the fit constant C for each trial. Now compare each of these values to the time constant of your circuit.
3. Resistors and capacitors are not marked with their exact values, but only approximate values with a tolerance. Determine the tolerance of the resistors and capacitors you are using. If there is a discrepancy between the two quantities compared in Question 2, can the tolerance values explain the difference?
4. What was the effect of reducing the resistance of the resistor on the way the capacitor discharged?
5. How would the graphs of your discharge graph look if you plotted the natural logarithm of the potential across the capacitor *vs.* time? Sketch a prediction. Show Run 1 (the first discharge of the capacitor) and hide the remaining runs. Click on the y-axis label and select $\ln(V)$. Click to see the new plot.
6. What is the significance of the slope of the plot of $\ln(V)$ *vs.* time for a capacitor discharge circuit?
7. Based upon the maximum potential difference of the fully charged capacitor, the Energy stored can be computed with $U = CV^2/2$. Compute the stored energy.

EXTENSIONS

1. What percentage of the initial potential remains after one time constant has passed? After two time constants? Three?
2. Use a Vernier Current Probe and Differential Voltage Probe to simultaneously measure the current through the resistor and the potential across the capacitor. How will they be related?
3. Instead of a resistor, use a small flashlight bulb. To light the bulb for a perceptible time, use a large capacitor (approximately 1 F). Collect data. Explain the shape of the graph.
4. Try different value resistors and capacitors and see how the capacitor discharge curves change.
5. Try two $10\ \mu F$ capacitors in parallel. Predict what will happen to the time constant. Repeat the discharge measurement and determine the time constant of the new circuit using a curve fit.
6. Try two $10\ \mu F$ capacitors in series. Predict what will happen to the time constant. Repeat the discharge measurement and determine the time constant for the new circuit using a curve fit.