

# ENERGY IN SIMPLE HARMONIC MOTION

## LAB MECH 17. COMP

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## INTRODUCTION

We can describe an oscillating mass in terms of its position, velocity, and acceleration as a function of time. We can also describe the system from an energy perspective. In this experiment, you will measure the position and velocity as a function of time for an oscillating mass and spring system, and from those data, plot the kinetic and potential energies of the system.

Energy is present in three forms for the mass and spring system. The mass  $m$ , with velocity  $v$ , can have kinetic energy  $KE$  given as

$$KE = \frac{1}{2}mv^2.$$

We note that the lower coils of the spring, which itself has mass, hang on the upper coils and progressively more so from bottom end coil to the top most coil. It turns out that the systems effective mass  $m$  is the sum of the hanging mass load plus one-third the mass of the spring.

The spring can hold elastic potential energy, or  $PE_{\text{elastic}}$ . We calculate  $PE_{\text{elastic}}$  by using

$$PE_{\text{elastic}} = \frac{1}{2}ky^2$$

where  $k$  is the spring constant and  $y$  is the extension or compression of the spring measured from the equilibrium position.

The mass and spring system also has gravitational potential energy ( $PE_{\text{gravitational}} = mgy$ ), but we do not have to include the gravitational potential energy term if we measure the spring length, that is  $y$ , from the hanging equilibrium position. We can then concentrate on the exchange of energy between kinetic energy and elastic potential energy.

If there are no other forces experienced by the system, then the principle of conservation of energy tells us that the sum  $\Delta KE + \Delta PE_{\text{elastic}} = 0$ , which we can test experimentally.

## PURPOSE

The purpose is to study and analyze the energy behavior of a hanging mass on a spring undergoing simple harmonic motion as it oscillates up and down, particularly testing for conservation of total energy as different forms of energy are exchanged.

## **MATERIALS**

Computer  
Vernier computer interface (Lab Pro)  
Logger *Pro*  
Vernier Motion Detector  
Spring, 1-10 N/m  
Slotted mass hanger

Slotted mass set, 50 g to 300 g in 50 g steps  
Ring stand and clamp/arm  
Wire basket  
Masking tape  
Balance

## **PRELIMINARY QUESTIONS**

1. Sketch a graph of the height *vs.* time for the mass on the spring as it oscillates up and down through one cycle. Mark on the graph the times where the mass moves the fastest and therefore has the greatest kinetic energy. Also mark the times when it moves most slowly and has the least kinetic energy.
2. On your sketch, label the times when the spring has its greatest elastic potential energy. Then mark the times when it has the least elastic potential energy.
3. From your graph of height *vs.* time, sketch velocity *vs.* time.
4. Sketch graphs of kinetic energy *vs.* time and elastic potential energy *vs.* time.

**PROCEDURE**

1. Obtain the mass of the spring and record in the Data Table. Mount the 200 g mass (include the mass of the mass hanger if used) and spring as shown in Figure 1. Connect the Motion Detector to the DIG/SONIC 1 channel of the interface. Position the Motion Detector directly below the hanging mass, taking care that no extraneous objects could send echoes back to the detector. Protect the Motion Detector by placing a wire basket over the detector. The mass should be about 50 cm above the detector when it is at rest. Using amplitudes of 10 cm or less will then keep the mass outside of the 40 cm minimum distance of the Motion Detector. If the observed graphical patterns contain considerable irregularities, further adjustments might be required in the position of the spring and hanging mass to avoid extraneous sonic signal reflections to the Motion Detector from such as a table edge or perhaps a person is too close to the sonic signal pattern.
2. Open the file “17a Energy in SHM” from the *Physics with Computers* folder.
3. Start the mass moving up and down by lifting it 5 cm and then releasing it. Take care that the mass is not swinging from side to side nor rotating. Click  to record position and velocity data. Sketch the position vs. time and the velocity vs. time graphs on the same sheet with the horizontal time axes vertically aligned so that time corresponds. Comment on: the related mass behavior with respect to position and velocity in time and also compare these results with your predictions under PRELIMINARY QUESTIONS.
4. To calculate the spring potential energy, it is necessary to measure the spring constant  $k$ . Hooke’s law states that the spring force is proportional to its extension from equilibrium, or  $F = -kx$ . You can apply a known force to the spring, to be balanced in magnitude by the spring force, by hanging a range of weights from the spring. The Motion Detector can then be used to measure the equilibrium positions. Open the experiment file “17b Energy in SHM.” Logger *Pro* is now set up to plot the applied weight vs. position.

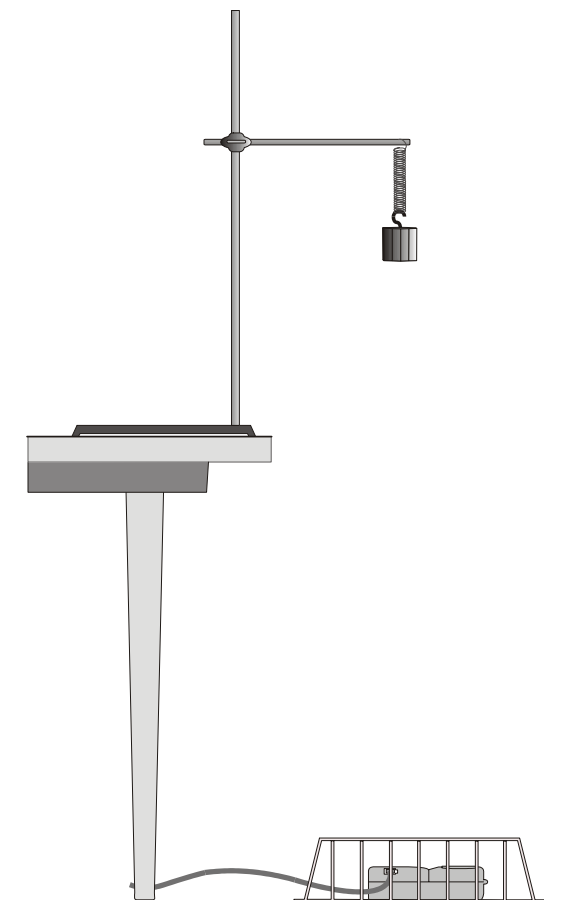


Figure 1

5. Click  to begin data collection. Hang a 50 g mass, or perhaps 100g or 150g (enough so that all the spring coils are separated), from the spring and allow the mass to hang motionless. Click  and enter, in the computer table space provided, the weight of the mass in newtons (N), that is,  $9.8 \text{ m/s}^2$  times the mass used in kg. Press ENTER to complete the entry. Now hang in increments of 50g a series of masses up to 300 g from the spring, recording, for each, the position and corresponding weight in N. When you are done, click  to end data collection.
6. Click on the Linear Fit button, , to fit a straight line to a linear portion of your data. The magnitude of the slope is the spring constant  $k$  in N/m. Record the value in the Data Table below.
7. Remove the 300 g mass and replace it with a 200 g mass for the following experiments.
8. Open the experiment file "17c Energy in SHM." In addition to plotting position and velocity, three new data columns have been set up in this experiment file (kinetic energy, elastic potential energy, and the sum of these two individual energies). You will need to modify the calculations for the energies. Click on Data from menu, Choose Column options, and select Kinetic Energy. In the Column Options table which is displayed note the kinetic energy equation and substitute your hanging mass plus one-third of that of the spring in kg instead of the 0.2 given in the equation. Then click . Similarly, again click on Data from menu, choose Column Options, and select Potential Energy. Now in the Column Options table displayed note the potential energy equation and substitute your determined spring constant value in N/m for the 5 in the equation and click on .
9. With the mass hanging from the spring and at rest, click  to zero the Motion Detector. From now on, all distances will be measured relative to this equilibrium position. When the mass moves closer to the detector, the position reported will be negative.
10. Start the mass oscillating in a vertical direction only, with an amplitude of about 10 cm by lifting it about 5 cm above equilibrium. Click  to gather position, velocity, and energy data.

## DATA TABLE

Spring constant	(N/m)
Spring Mass	(kg)

Position	Time (s)	Height (m)	Velocity (m/s)	PE (J)	KE (J)	TE (J)
Top of path						
Between top and equilibrium						
Equilibrium						
Between equilibrium and bottom						
Bottom of path						

## ANALYSIS

1. As you work through the following ANALYSIS, fill in the Data Table with appropriate values for the positions, of the moving mass, as indicated by the labels.
2. Click on the y-axis label of the velocity graph to choose another column for plotting. Click on “More” to see all of the columns. Uncheck the velocity column and select the kinetic energy and potential energy columns. Click  to display the new plot(s).
3. Compare your two energy plots to the sketches you made earlier. Be sure you compare to a single cycle beginning at the same point in the motion as your predictions. You may need to increase the energy scale to well observe the plots by selecting Analyze from menu and click on Zoom Graph In Ctrl ++. Comment on any differences.
4. If mechanical energy is conserved in this system, how should the sum of the kinetic and potential energies vary with time? State in words your prediction of the sum of energies as a function of time.
5. Check your prediction. Click on the y-axis label of the position graph to choose another column for plotting. Click on “More” and select the total energy column in addition to the other energy columns (that is click on the little squares so that Kinetic Energy (KE), Potential Energy (PE), and Total Energy (TE) will be displayed). Click  to draw the new plot. You might wish to enlarge graph displays – note 3. above.
6. From the shape of the total energy vs. time plot, what can you conclude about the conservation of mechanical energy in your mass and spring system?

## EXTENSIONS

1. In the introduction, we claimed that the gravitational potential energy could be ignored *if* the displacement used in the elastic potential energy was measured from the hanging equilibrium position. First write the total mechanical energy (kinetic, gravitational potential, and elastic potential energy) in terms of a coordinate system, position measured upward and labeled  $y$ , whose origin is located at the bottom of the relaxed spring of constant  $k$  (no force applied). Then determine the equilibrium position  $s$  when a mass  $m$  is suspended from the spring. This will be the new origin for a coordinate system with position labeled  $h$ . Write a new expression for total energy in terms of  $h$ . Show that when the energy is written in terms of  $h$  rather than  $y$ , the gravitational potential energy term cancels out.
2. If a non-conservative force such as air resistance becomes important, the graph of total energy vs. time will change. Predict how the graph would look, then tape an index card to the bottom of your hanging mass. Take energy data again and compare to your prediction.
3. The energies involved in a swinging pendulum can be investigated in a similar manner to a mass on a spring. From the lateral position of the pendulum bob, find the bob’s gravitational potential energy. Perform the experiment, measuring the horizontal position of the bob with a Motion Detector.
4. Set up a laboratory cart or a glider on an air track so it oscillates back and forth horizontally between two springs. Record its position as a function of time with a Motion Detector. Investigate the conservation of energy in this system. Be sure you consider the elastic potential energy in both springs.