Possible Exam 1 Questions

1. Create a piecewise function (consisting of only TWO rational functions) that has exactly 3 discontinuities, a removable discontinuity at \( x = -3 \), a jump discontinuity at \( x = 1 \), and an infinite discontinuity at \( x = 5 \). Then verify each type of discontinuity.

2. For the function created in question 1, add a constant to the second rational function to make the function continuous at \( x = 1 \). Then verify that the function is continuous at \( x = 1 \) using the three conditions of continuity.

3. A function can be continuous at \( x = 2 \) without \( f'(2) \) existing. Give two examples of functions having different derivatives that have this property.

4. Find \( y' \) for each of the following:
   
   a. \( y = \sin x + \cos x + \tan x + \cot x + \sec x + \csc x \)
   
   b. \( y = \left( x^2 + 5x + 2 \right) \left( \frac{\sin(x^2 + 1)}{x^3 + 5} \right)^7 \)
   
   c. \( y^4 - x^4 = 11 \) (Also find \( y'' \) for part c.)

5. Evaluate each of the following limits
   
   a. \( \lim_{x \to 2^+} \frac{x^4 - 2x^2 + 3x - 14}{x - 2} \)
   
   b. \( \lim_{h \to 0} \frac{(x + h)^4 - x^4}{h} \)
   
   c. \( \lim_{x \to 3} \frac{\sqrt{5x + 1} - 4}{x - 3} \)
   
   d. \( \lim_{x \to 0} \frac{x \sin(7x) \cos(5x)}{\tan^2(2x)} \)

6. Use the quotient rule to find the derivative of \( \cot x \).

7. Sketch a graph of a function satisfying all of the following:
   
   a. \( \lim_{x \to 1} f(x) = 4 \)
   
   b. \( \lim_{x \to 3} f(x) = 2 \)
   
   c. \( \lim_{x \to 3} f(x) = 5 \)
   
   d. \( f(1) = 2 \)
   
   e. \( \lim_{x \to 5} f(x) = \infty \)
   
   f. \( \lim_{x \to 5} f(x) = -\infty \)
   
   g. \( f(3) \) d.n.e.

8. Use the limit definition of the derivative to find the derivative of
   
   a. \( f(x) = a\sqrt{x} \)
   
   b. \( f(x) = 4x^2 - 5x + 2 \)

9. Explain how the intermediate value theorem can be used to find the solution to \( x^3 - 4x + 1 = 0 \).

10. Find the equation of the tangent line to the graph of \( y = \sec^3 x + 1 \) when \( x = \frac{\pi}{3} \).