The Search for Extremal Graphs

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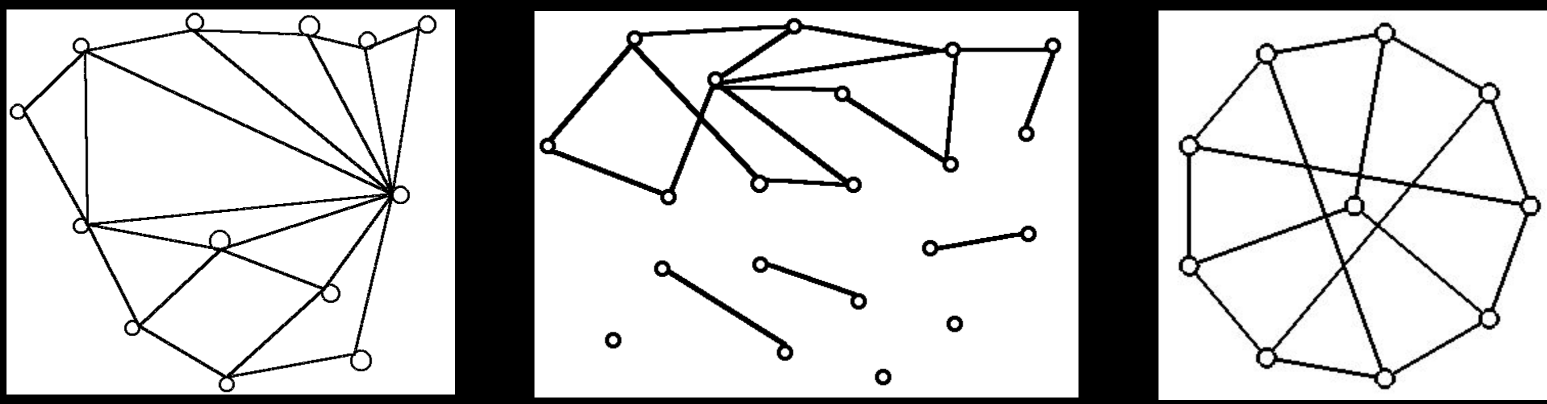
Advised by Dr. David Offner

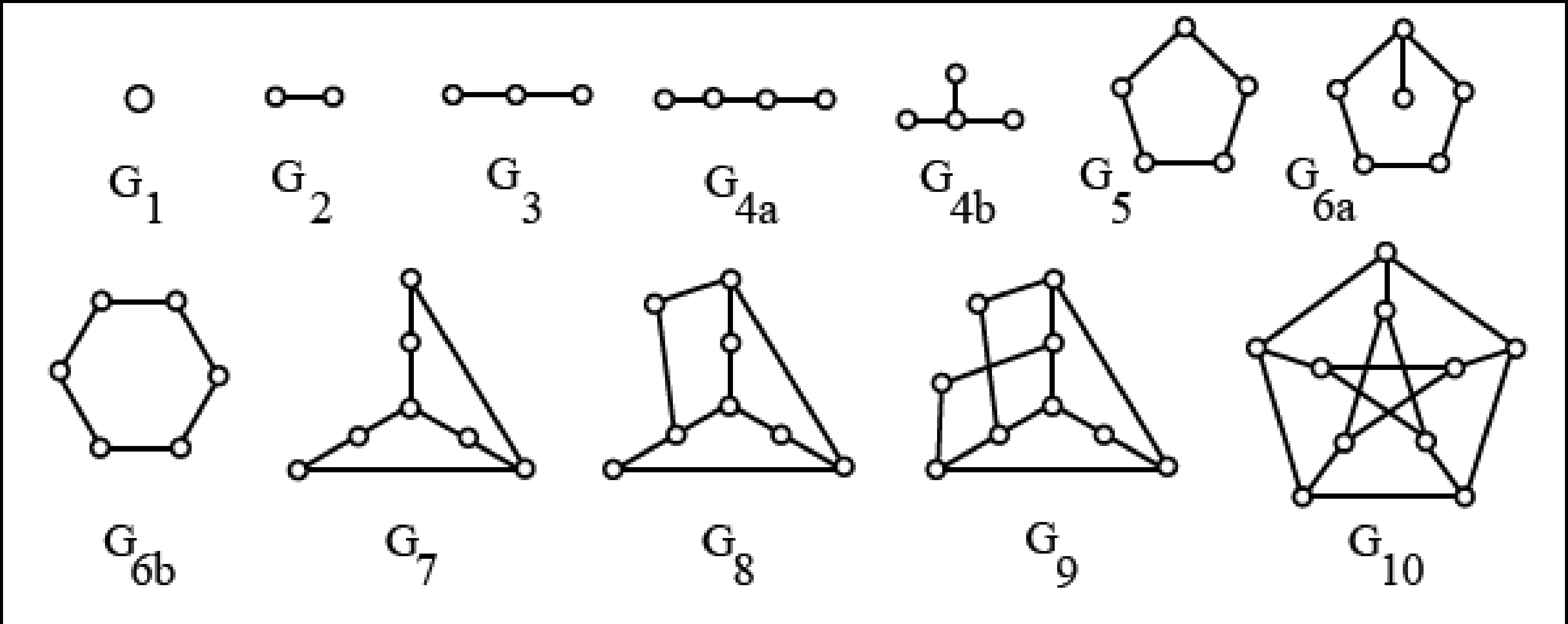
What is a graph?



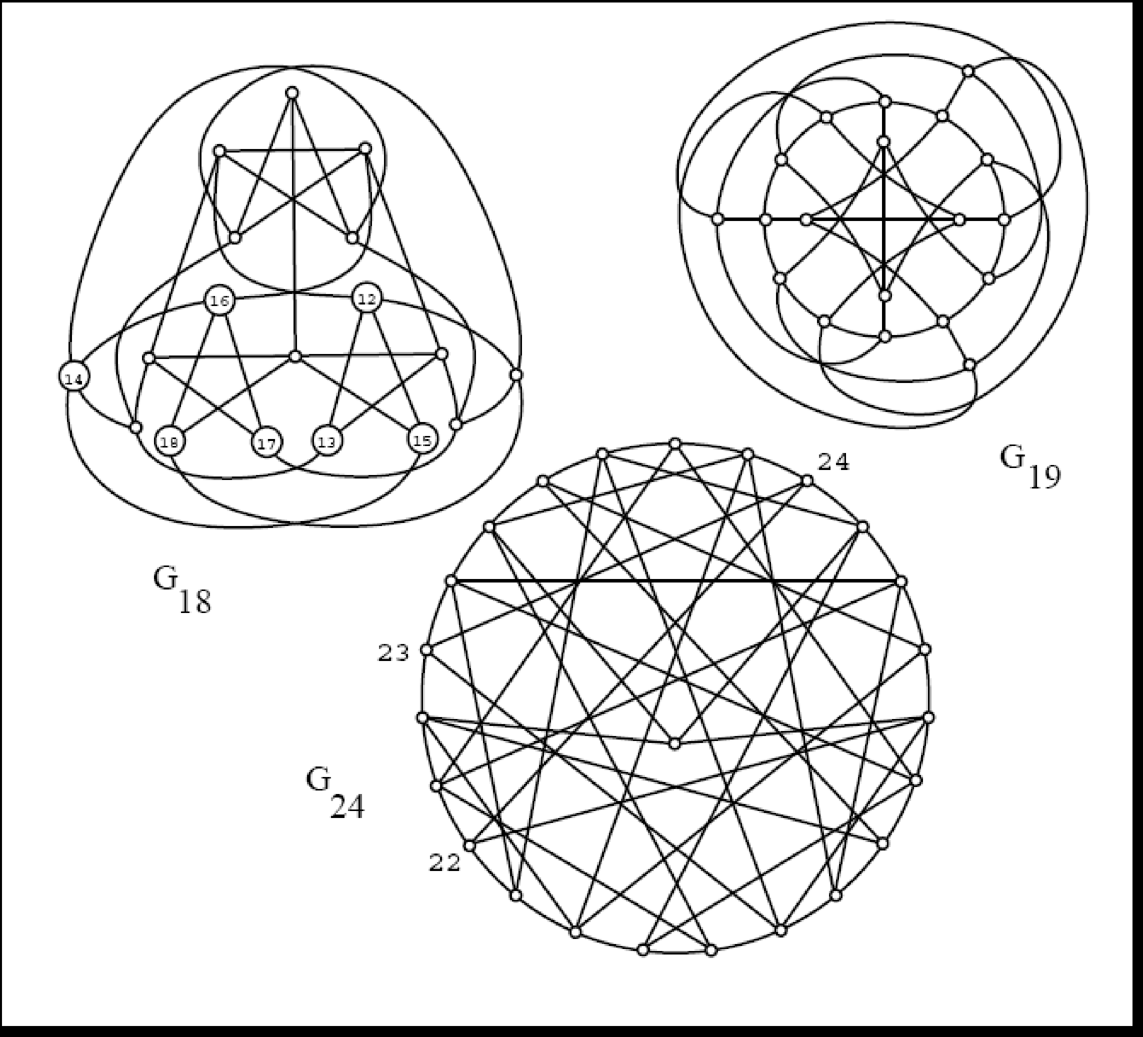
- e = the number of edges
- v = the number of vertices
- C_3 is a 3-cycle
- C_4 is a 4-cycle

The Search for Extremal Graphs

Our Question: Given a graph with v vertices, what is greatest number of edges, f(v), the graph can have without containing a 3 or 4-cycle?



Extremal Graphs of Higher Order



Previous Work

- Conjecture:

$$f(v) = [1/2 + o(1)]^{\frac{3}{2}} v^{\frac{3}{2}}$$

(Erdős 1975)

- Known theoretical bounds:
  - For all v: $f(v) \leq \frac{1}{2} v \sqrt{v-1}$
  - For infinitely many v: $f(v) \geq \left(\frac{1}{2}\right)^{\frac{3}{2}} v^{\frac{3}{2}}$

(Garnick, Kwong, Lazebnik, 1993)

Previous Work

$f(v)$	0	1	2	3	4	5	6	7	8	9
0	**0**	**0**	**1**	**2**	**3**	**5**	**6**	**8**	**10**	**12**
10	**15**	**16**	**18**	**21**	**23**	**26**	**28**	**31**	**34**	**38**
20	**41**	**44**	**47**	**50**	**54**	**57**	**61**	**65**	**68**	**72**
30	**76**	80	85	87	90	94	99	104	109	114
40	120	124	129	134	139	144	150	156	162	168
50	**175**	176	178	181	185	188	192	195	199	203
60	207	212	216	221	226	231	235	240	245	250
70	255	260	265	270	275	280	285	291	296	301
80	306	311	317	323	329	334	340	346	352	357
90	363	368	374	379	385	391	398	404	410	416
100	422	428	434	440	446	452	458	464	470	476
110	483	489	495	501	508	514	520	526	532	538
120	544	551	558	565	571	578	584	590	596	603
130	610	617	623	630	637	644	651	658	665	672
140	679	686	693	700	707	714	721	728	735	742
150	749	756	763	770	777	784	791	798	805	812
160	819	826	834	841	849	856	863	871	878	886
170	893	901	909	917	925	933	941	948	956	963
180	971	979	986	994	1001	1009	1017	1025	1033	1041
190	1049	1057	1065	1073	1081	1089	1097	1105	1113	1121
200	1129									

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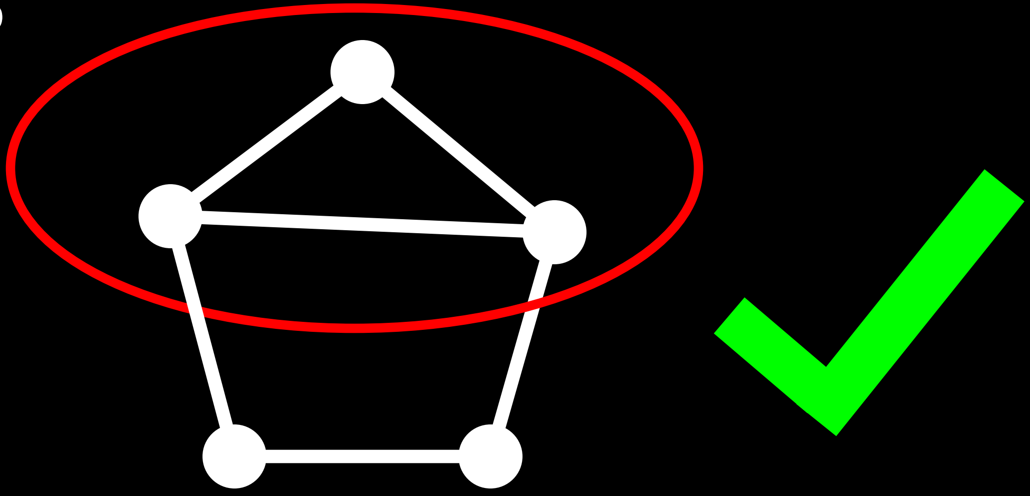
Hills

Computer Search for Extremal Graphs

- Hill Climbing
  - Most basic method
  - No backtracking (you are never "descending")
  - Results match the results found by Garnick et. al. for v < 36



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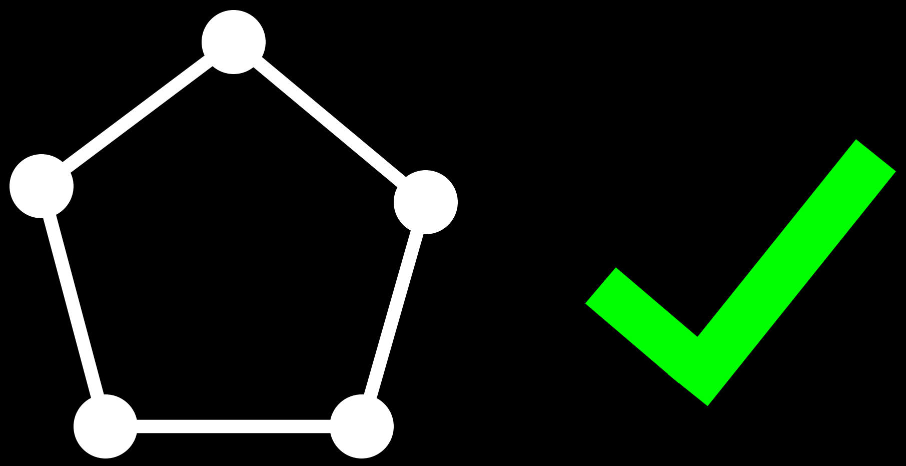
More Advanced Search

- Hill Climbing with checking
  - Builds quickly
  - Results similar to Hill Climbing, but faster



Previous Work

$f(v)$	0	1	2	3	4	5	6	7	8	9
0	**0**	**0**	**1**	**2**	**3**	**5**	**6**	**8**	**10**	**12**
10	**15**	**16**	**18**	**21**	**23**	**26**	**28**	**31**	**34**	**38**
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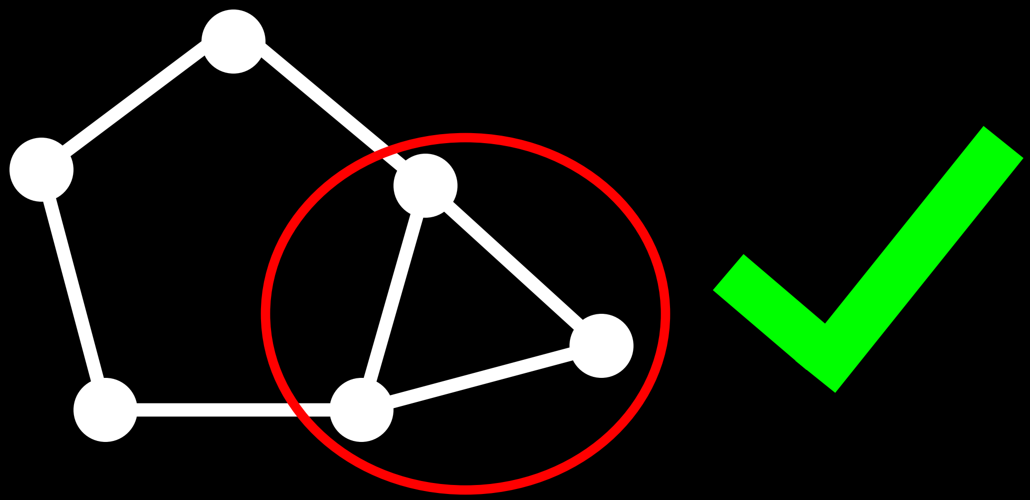
Our Best Method

- Hill Building
  - Builds on known extremal graphs
  - Surpasses the results found by Garnick, Kwong, and Lazebnik for v = 45, 55, 57, 58, 59, 60, 75, 76, 81, 82, 85, 86, 87



Our Best Method

f(v)	0		1		2		3		4		5		6		7		8		9	
0	0	0	0	0	1	1	2	2	3	3	5	5	6	6	8	8	10	10	12	12
10	15	15	16	16	18	18	21	21	23	23	26	26	28	28	31	31	34	34	38	38
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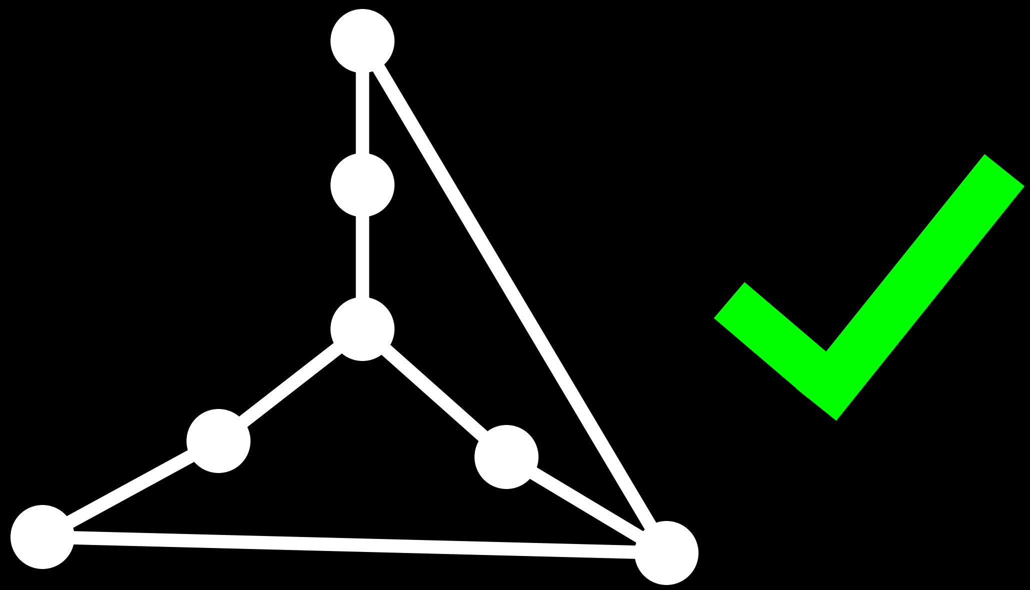
80	306	306	311	312	317	318	323	323	329	329	334	335	340	341	346	347	352	352	357	357
90	363	363	368	368	374	374	379	379	385	385	391	391	398	398	404	404	410	410	416	416

Another Trick

- Hill Descending
  - Steps down from known extremal graph



Future Areas of Exploration

- Improve computer results
  - Improve current Hill Climbing and Building Strategies
  - Implementation new search strategies
- Try to prove a given graph is extremal
- Apply methods to other problems