MTH131 - Applied Calculus - Spring 2015
Lab 9 – Indefinite and Definite Integrals
SOLUTIONS

1. (a) $\frac{d}{dx}x^7 = 7x^6$

   $\int x^7\,dx = \frac{1}{8}x^8 + C$

   (b) $\frac{d}{dx}\sqrt[3]{x} = \frac{1}{3}x^{-2/3}$

   $\int \sqrt[3]{x}\,dx = \frac{3}{4}x^{4/3} + C$

   (c) $\frac{d}{dx}\frac{1}{\sqrt{z}} = \frac{d}{dx}z^{-1/2} = -\frac{1}{2}z^{-3/2}$

   $\int \frac{1}{\sqrt{z}}\,dz = \int z^{-1/2}\,dz = 2z^{1/2} + C$

   (d) $\frac{d}{dx}(15x^4 - 12x + 3) = 60x^3 - 12$

   $\int (15x^4 - 12x + 3)\,dx = 3x^5 - 6x^2 + 3x + C$

   (e) $\frac{d}{dx}(\sqrt{t}+1)^2 = 2(\sqrt{t}+1)\left(\frac{1}{2}t^{-1/2}\right) = (\sqrt{t}+1)t^{-1/2}$

   $\int (\sqrt{t}+1)^2\,dt = \int (t + 2\sqrt{t} + 1)\,dt = \frac{1}{2}t^2 + \frac{4}{3}t^{3/2} + t + C$

   (f) $\frac{d}{dx}e^{4x} = 4e^{4x}$

   $\int e^{4x}\,dx = \frac{1}{4}e^{4x} + C$

   (g) $\frac{d}{dx}24e^{-4u/3} = -32e^{-4u/3}$

   $\int 24e^{-4u/3}\,du = -18e^{-4u/3} + C$

   (h) $\frac{d}{dx}\frac{5}{2x} = -\frac{5}{2x^2}$

   $\int \frac{5}{2x}\,dx = \frac{5}{2}\ln|x| + C$

   (i) $\frac{d}{dx}\frac{(x+1)(x-3)}{x^3} = \frac{d}{dx}\left(\frac{1}{x} - \frac{2}{x^2} - \frac{3}{x^3}\right) = -\frac{1}{x^2} + \frac{4}{x^3} + \frac{9}{x^4}$

   $\int \frac{(x+1)(x-3)}{x^3}\,dx = \int \left(\frac{1}{x} - \frac{2}{x^2} - \frac{3}{x^3}\right)\,dx = \ln|x| + \frac{2}{x} + \frac{3}{2x^2} + C$

2. The integral of the rate of change of the average surface temperature gives us a formula $f(t)$ for the average surface temperature:

$$f(t) = \int 0.014t^{0.4}\,dt = \frac{0.014}{1.4}t^{1.4} + C = 0.01t^{1.4} + C$$

To determine the constant C, we are told that the average surface temperature at $t = 0$ is 57.8, so

$$\begin{aligned} f(0) = 0.01(0)^{1.4} + C &= 57.8, \\ C &= 57.8. \end{aligned}$$

The complete formula is therefore $f(t) = 0.01t^{1.4} + 57.8$.

Using this formula, the average surface temperature in the year 2020 is predicted to be $f(20) = 0.01(20)^{1.4} + 57.8 = 58.5$

3. To solve this problem we need a height formula $h(t)$ which gives us the height of the rocket at any time t. Our first step is determining the indefinite integral of the velocify function

$$\begin{aligned} h(t) &= \int 200(1 - e^{-0.5t})\, dt \\ h(t) &= 200 \int 1 - e^{-0.5t})\, dt \\ &= 200 \left(t - \frac{1}{-0.5} e^{-0.5t} \right) + C \\ &= 200 \left(t + 2e^{-0.5t} \right) + C \end{aligned}$$

At $t = 0$ we are told that $h(0) = 150$, so

$$\begin{aligned} h(0) = 200 \left(0 + 2e^{-0.5(0)} \right) + C &= 150 \\ 200(0 + 2) + C &= 150 \\ 400 + C &= 150 \\ C &= -250 \end{aligned}$$

The final formula for the height of the rocket becomes $h(t) = 200 \left(t + 2e^{-0.5t} \right) - 250$. At $t = 2$, the height of the rocket is $h(2) = 200(2 + 2e^{-0.5(2)}) - 250 = 297.15$ miles

4. To solve this problem we need a distance formula $d(t)$ which gives us the distance at any time t. Our first step is determining the indefinite integral of the velocity function

$$\begin{aligned} d(t) &= \int v(t)\, dt, \\ &= \int (6 - 2t)\, dt, \\ &= 6t - t^2 + C. \end{aligned}$$

We determine the constant C based on the initial conditions for each problem

   (a) If the ball is thrown from the ground at time $t = 0$, that means $d(0) = 0$. Using the equation above this means $6(0) - (0)^2 + C = 0$ or $C = 0$. Thus $d(t) = 6t - t^2 = t(6 - t)$. To determine when the ball will hit the ground again, we find a value for $t > 0$ so that $d(t) = 0$. This occurs when $t = 6$.

   (b) If the ball is thrown from a height of 27 feet at time $t = 0$, that means $d(0) = 27$. Using the equation above this means $6(0) - (0)^2 + C = 27$ or $C = 27$. Thus $d(t) = 6t - t^2 - 27 = (t - 9)(-t + 3)$. To determine when the ball will hit the ground again, we find a value for $t > 0$ so that $d(t) = 0$. This occurs when $t = 9$ (NOTE: $t = -3$ also satisfies $d(t) = 0$, but does not satisfy the condition that $t > 0$).

5. (a) $\int_1^2 (12-3x^2)\,dx = (12x-x^3)|_1^2 = (12(2)-2^3)-(12(1)-1^3) = 16-11 = 5$

   (b) $\int_1^4 \frac{1}{x^3}\,dx = \frac{-1}{2x^2}\Big|_1^4 = \frac{-1}{2(4^2)} - \left(\frac{-1}{2(1^2)}\right) = \frac{-1}{32} - \frac{-1}{2} = \frac{15}{32}$

   (c) $\int_1^4 \frac{1}{x}\,dx = \ln x|_1^4 = \ln 4 - \ln 1 = \ln 4$

   (d) $\int_{-1}^2 5e^x\,dx = 5e^x|_{-1}^2 = 5e^2 - 5e^{-1}$

   (e) $\int_1^2 \frac{(x+1)^2}{x}\,dx = \int_1^2 \frac{x^2+2x+1}{x}\,dx = \int_1^2 x+2+x^{-1}\,dx = \left(\frac{1}{2}x^2+2x+\ln x\right)\Big|_1^2$
   $= \left(\frac{1}{2}2^2+2(2)+\ln 2\right) - \left(\frac{1}{2}1^2+2(1)+\ln 1\right) = 6+\ln 2 - \frac{5}{2} = \frac{7}{2}+\ln 2$

6. (a) $-x^2+x+6 = -(x-3)(x+2)$, so the two zeros are $x=3$ and $x=-2$.

   (b) $\int_{-2}^3 -x^2+x+6\,dx = -\frac{1}{3}x^3+\frac{1}{2}x^2+6x\Big|_{-2}^3 = -\frac{1}{3}3^3+\frac{1}{2}3^2+6(3)-\left(-\frac{1}{3}(-2)^3+\frac{1}{2}(-2)^2+6(-2)\right) = \frac{125}{6}.$

7. Here we need a definite integral to convert a rate of change (velocity) to a net overall change in quantity (distance). Using the given function and limits 2 to 3 we have

$$\begin{aligned}\int_2^3 200(1-e^{-0.5t})\,dt &= 200\int_2^3 (1-e^{-0.5t})\,dt\\ &= 200\left(t-\frac{1}{-0.5}e^{-0.5t}\right)\Big|_2^3\\ &= \left(200t+400e^{-0.5t}\right)\Big|_2^3\\ &= \left(200(3)+400e^{-0.5(3)}\right)-\left(200(2)+400e^{-0.5(2)}\right)\\ &= 689.25-547.15 = 142.10.\end{aligned}$$

For the limits 9 to 10 we have

$$\begin{aligned}\int_9^{10} 200(1-e^{-0.5t})\,dx &= \left(200x+400e^{-0.5t}\right)\Big|_9^{10}\\ &= \left(200(10)+400e^{-0.5(10)}\right)-\left(200(9)+400e^{-0.5(9)}\right)\\ &= 2002.70-1804.44 = 198.25.\end{aligned}$$

8. (a) When x is in the range $[0,2]$, e^x is the "upper" function, so the area between the curves is given by

$$\int_0^2 (e^x-e^{-x})\,dx = (e^x+e^{-x})|_0^2 = (e^2+e^{-2})-(e^0+e^{-0}) = e^2+e^{-2}-2 \approx 5.52$$

   (b) We proceed as above with different limits of integration

$$\int_0^{\ln 2} (e^x-e^{-x})\,dx = (e^x+e^{-x})|_0^{\ln 2} = (e^{\ln 2}+e^{-\ln 2})-(e^0+e^{-0}) = (2+\frac{1}{2})-2 = \frac{1}{2}.$$