

MTH131 Applied Calculus – Spring 2016
Lab 4 – SOLUTIONS

1. (a) $f'(x) = 5x^4 - 9x^2 + 24x$
 $f''(x) = 20x^3 - 18x + 24$
- (b) Rewrite $f(x) = x^{7/2}$:
 $f'(x) = (7/2)x^{5/2}$
 $f''(x) = (7/2)(5/2)x^{3/2} = (35/4)x^{3/2}$
- (c) $f'(x) = 2x(x^2 + 2) + (x^2 - 3)(2x) = 4x^3 - 2x$
 $f''(x) = 12x^2 - 2$
- (d) Rewrite $f(x) = x^{-1} + x^{-3}$:
 $f'(x) = -x^{-2} - 3x^{-4} = -\frac{1}{x^2} - \frac{3}{x^4}$
 $f''(x) = 2x^{-3} + 12x^{-5} = \frac{2}{x^3} + \frac{12}{x^5}$

2. (a) $f'(x) = 3(2x^2 - 8x + 7)^2(4x - 8)$
- (b) $f'(x) = \frac{1}{2}(x^5 - 3x^4 + x)^{-1/2}(5x^4 - 12x^3 + 1) = \frac{5x^4 - 12x^3 + 1}{2\sqrt{x^5 - 3x^4 + x}}$
- (c) $f'(x) = 4(1 - x)^3(-1) = -4(1 - x)^3$
- (d) $f'(x) = 15x^2(10 - 2x)^2 + 5x^3(2(10 - 2x))(-2) = 15x^2(10 - 2x)^2 - 20x^3(10 - 2x)$
 $= 5x^2(10 - 2x)[3(10 - 2x) - 4x]$
 $= 5x^2(10 - 2x)(30 - 10x)$

Note: On exams and quizzes, you could have stopped after the first line above.

- (e) $f'(x) = \frac{(1 - 4x)^2 6x - 2(1 - 4x)(-4)(3x^2 + 5)}{(1 - 4x)^4} = \frac{6(1 - 4x)x + 8(3x^2 + 5)}{(1 - 4x)^3} = \frac{6x + 40}{(1 - 4x)^3}$
- (f) $f'(x) = \frac{1}{2} \left(1 + 2(1 + 3x)^{1/2}\right)^{-1/2} \frac{1}{2} \left(2(1 + 3x)^{-1/2}(3)\right) = \frac{3}{2\sqrt{1 + 2\sqrt{1 + 3x}}\sqrt{1 + 3x}}$

Note that here we had to use the Generalized Power Rule twice.

3. (a) We obtain formulas for vertical and horizontal velocity by taking the derivatives of the vertical and horizontal distance formulas:

$$\begin{aligned}v_v(t) &= 800 - 32t \\v_h(t) &= 600\end{aligned}$$

Likewise, we obtain formulas for vertical and horizontal acceleration by taking the derivatives of the vertical and horizontal velocity formulas:

$$\begin{aligned}a_v(t) &= -32 \\a_h(t) &= 0\end{aligned}$$

At $t = 5$, the vertical and horizontal velocities are $v_v(5) = 640$ and $v_h(5) = 600$; at $t = 10$, the vertical and horizontal velocities are $v_v(10) = 480$ and $v_h(10) = 600$. The vertical and horizontal accelerations at both times are -32 and 0 , respectively.

(b) When the cannonball hits the ground, the vertical height is 0. This occurs when

$$\begin{aligned}d_v(t) &= 800t - 16t^2 = 0, \\t(800 - 16t) &= 0, \\t &= 0, 800/16 = 50.\end{aligned}$$

The horizontal distance at this point is $d_h(50) = 30000$.

(c) When the cannonball is at its highest point, the vertical velocity is 0. This occurs when

$$\begin{aligned}v_v(t) &= 800 - 32t = 0, \\t &= 800/32 = 25.\end{aligned}$$

The horizontal distance at this point is $d_h(25) = 15000$.

4. (a) $f'(x) = 3x^2 - 48$. Setting this equal to 0 leads to $x = 4, -4$ as the two critical numbers.
 (b) $f'(x) = 2(2x - 8)(2) = 8x - 32$. This is 0 when $x = 4$, which is the only critical number.
 (c) $f'(x) = 2(2x^2 - 8)(4x) = 16x^3 - 64x = 16x(x^2 - 4)$ which is 0 when $x = 0, 2$ or -2 .
 (d) $f'(x) = 2(3x+6)(3)(x-2) + (3x+6)^2(1) = (3x+6)(6(x-2) + 3x+6) = (3x+6)(9x-6)$. This is 0 when $x = -2, 2/3$.

(e) $f'(x) = \frac{(x+1)(x-5)(0) - 1[1(x-5) + (x+1)(1)]}{[(x+1)(x-5)]^2}$ or $\frac{-(2x-4)}{[(x+1)(x-5)]^2}$. This is zero when the numerator is 0, i.e. when $x = 2$. Two other critical numbers are $x = -1$ and $x = 5$, where the derivative is undefined.

5. (a) $f''(x) = 6x$. Setting this equal to 0 leads to $x = 0$ as the inflection point.
 (b) $f''(x) = 8$. Since this can never be 0, there are no inflection points for this function.
 (c) $f''(x) = 48x^2 - 64$ which is 0 when $x = \pm\sqrt{4/3}$.
 (d) $f''(x) = 3(9x - 6) + (3x + 6)9 = 54x + 36$. This is 0 when $x = -2/3$.

6. From Problem 1:

$$\begin{aligned}(a) \quad f^{(3)}(x) &= 60x^2 - 18 \\(b) \quad f^{(3)}(x) &= (35/4)(3/2)x^{1/2} = (105/8)\sqrt{x} \\(c) \quad f^{(3)}(x) &= 24x \\(d) \quad f^{(3)}(x) &= -6x^{-4} - 60x^{-6} = -\frac{6}{x^4} - \frac{60}{x^6}\end{aligned}$$

From Problem 2:

$$\begin{aligned}(a) \quad f''(x) &= 6(2x^2 - 8x + 7)(4x - 8)(4x - 8) + 3(2x^2 - 8x + 7)^2(4) = 6(2x^2 - 8x + 7)(4x - 8)^2 + 12(2x^2 - 8x + 7)^2 \\(b) \quad f''(x) &= -\frac{1}{4}(x^5 - 3x^4 + x)^{-3/2}(5x^4 - 12x^3 + 1)^2 + \frac{1}{2}(x^5 - 3x^4 + x)^{-1/2}(20x^3 - 36x^2) \\(c) \quad f''(x) &= -4(3)(1-x)^2(-1) = 12(1-x)^2\end{aligned}$$