

Circular Motion and Centripetal Force

LAB MECH 12. COMP

From *Physics with Computers*, Vernier Software & Technology, 2003.

INTRODUCTION

With circular motion there is a force directed perpendicular to the motion and towards the center that is common to our awareness. This force is at work in an automobile negotiating a curve, skaters in cracking-the-whip, and that of the moon and earth revolving around each other about the gravitational balance (barycenter) between them.

Such a force is forthrightly felt in the tug or tension T by the string when swinging a mass object in a “horizontal” circle over ones head and is called a centripetal force. Velocity being a vector quantity involves acceleration when undergoing a change whether in magnitude, in direction, or in both magnitude and direction.

For acceleration to occur requires force, which according to Newton’s second law of motion the force F required for the acceleration a , that is, to change the velocity, is equal to the mass m undergoing the velocity change times the acceleration:

$$F = ma. \quad (1)$$

Now for circular motion the acceleration involves change of direction continuing perpendicular to the direction of the motion, called the radial or centripetal acceleration, and so, is directed towards the center of a circle. In taking the limit of this change of motion direction, to a point on the circle, this radial acceleration a_{rad} can be expressed as:

$$a_{\text{rad}} = v^2/r \quad (2)$$

where v is the magnitude of the tangential circular velocity or speed and r is the radius of curvature, that is, the radius of the circular motion. The radial Force F_{rad} then can be expressed as

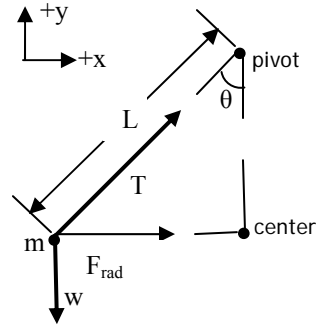
$$F_{\text{rad}} = mv^2/r. \quad (3)$$

The experimental arrangement for the study of this motion involves a mass object suspended from a pivot point on a string and “swung” in such a manner that the object swings in a circle in a horizontal plane creating a tension in the string that involves the centripetal force.

The figure to the right is the free-body diagram of mass m undergoing circular motion. Considering the motion to take place in the x -axis plane, the radial force would be

$$F_{\text{rad}} = T \sin \theta \quad (4)$$

where θ , as indicated in the diagram, is the direction that tension acts on the mass with respect to the vertical. Substituting in terms of equation (3), we obtain



$$T \sin \theta = mv^2/r. \quad (5)$$

Noting that mass m travels the distance $2\pi r$ around the circle in a cycle or period P , results in an object speed of

$$v = 2\pi r/P. \quad (6)$$

Substituting this relationship for v in equation (5) gives

$$T \sin \theta = m 4 \pi^2 r/P^2. \quad (7)$$

Noting from the diagram that radius

$$r = L \sin \theta, \quad (8)$$

which then by equation (7) gives the relationship

$$T = m 4 \pi^2 L/P^2. \quad (9)$$

One observes that this relationship lends itself conveniently to experiment since the angle θ while implicitly involved, does not need to be measured in determining tension T . Mass m and length L are easily obtained. Experimentally, tension T can be measured with a force sensor and period P gotten from counting/timing a number of swing cycles.

To compute the actual centripetal force F_{rad} , from the free-body diagram we note that

$$\theta = \cos^{-1} w/T \quad (10)$$

and then by equation (4),

$$F_{\text{rad}} = T \sin(\cos^{-1} w/T). \quad (11)$$

PURPOSE

The purpose of this experiment is to sense and measure the radial or centripetal force acting perpendicular to the motion of an object undergoing circular motion.

EQUIPMENT/MATERIALS

Laptop computer with Logger Pro 3.3
LabPro with AC adapter
LabPro → computer cable
Vernier Force Sensor

force sensor mounting
mass with string
balance
meter stick

PRELIMINARY QUESTIONS

1. If the string breaks when an object is undergoing the circular motion, describe its subsequential motion.
2. In the orbital motion of the planets about the sun, does the distance from the sun depend on the planet's mass – explain?
3. Why when approaching a curve on a highway, signs are posted to reduce speed?

PROCEDURE

1. Connect the Vernier Force Sensor to CH1 of the LabPro.
2. Attach the mass with string to the force sensor and place the force sensor in the special mounting.
3. Open the file in the Experiment 11 (Newton's Third Law) folder of *Physics with Computers*. While Experiment 11 was not designed for the experiment at hand, with some adaptation it lends itself to the desired treatment. In bringing up this file for the "Sensor Confirmation" display, click on "OK" even though using one force sensor rather than the two needed for Newton's Third Law. Two force vs time coordinate axis systems, one over the other, will be displayed. Since only one force sensor is involved in this experiment and connected to LabPro CH1, only the upper displayed coordinates are germane to this experiment.
4. To assure that the force sensor reads accurately, proceed to calibrate it as follows:
 - a. Choose Calibrate from the Experiment menu. Select CH1: Dual Range Force. Click on the button.
 - b. Remove all force from the first sensor and hold it vertically with the hook pointed down. Enter a **0** (zero) in the Value 1 field, and after the reading shown for Reading 1 is stable, click . This defines the zero force condition.
 - c. Hang the 500 g mass from the sensor. This applies a force of 4.9 N. Enter **4.9** in the Value 2 field, and after the reading shown for Reading 2 is stable, then click . Click to complete the calibration of the first Force Sensor.
5. Attach to the force sensor arrangement the provided mass m_1 with the string attached having a length L_1 of about 0.55m.
6. Holding the arrangement with axis vertical and with ones arms extended to allow space for the circular swinging motion, provide the motion needed that the hanging mass swings with approximately constant circular motion in a horizontal plane.
7. Click to take a trial run of data. With seemingly satisfactory data as displayed on the force vs. time plot, click to begin collecting data for doing the experiment.

DATA TABLE Tension/Centripetal Force

Mass, $m_1 =$ (kg), *Length, $L_1 =$ (m)						
Trial	Number of cycles	Cycles time span(s)	Period (s)	Calculated Tension, T (N)	Experiment Tension, T (N)	**Centripetal Force, F_{rad} (N)
1						
2						
3						


Mass, $m_1 =$ (kg), *Length, $L_2 =$ (m)						
Trial	Number of cycles	Cycles time span(s)	Period (s)	Calculated Tension, T (N)	Experiment Tension, T (N)	**Centripetal Force, F_{rad} (N)
1						
2						
3						

Mass, $m_2 =$ (kg), *Length, $L_3 =$ (m) (make $L_3 \approx L_1$)						
Trial	Number of cycles	Cycles time span(s)	Period (s)	Calculated Tension, T (N)	Experiment Tension, T (N)	**Centripetal Force, F_{rad} (N)
1						
2						
3						

*L, length, measured from pivot to middle of ball

**Based upon calculated T value

ANALYSIS

1. Note the DATA TABLE provided to enter your various data and results.
2. Typically a somewhat repeating/varying force pattern is observed so that an integral number of cycles can be counted off for a region scanned. With the cursor, scan an integral number of cycles of revolution which display a rather overall uniform force and click on the statistics button  to obtain the mean experimental tension T . With the cursor obtain the time limits for this region and thereby compute the period P for one cycle.
3. Take the mass m and length L measurements and then for each trial with equation (9), calculate tension T . Also, with equation (11) calculate the corresponding centripetal force based on the calculated tension.
4. Repeat the procedure for m_1 with a shorter length L_2 so as to complete the middle part of the DATA TABLE. Then, similarly, with different mass m_2 and a length L_3 of approximately that of L_1 complete the bottom part of the DATA TABLE.
5. Compare the calculated and experimental values of T . Discuss factors that might affect the results not accounted for in this treatment.

EXTENSION

Study and discuss the effects of different speeds and also for different m and L values.